Journal of Applied Mechanics and Technical Physics, Vol. 39, No. 3, 1998

SHOCK-WAVE STRUCTURE IN A DISPERSING PLASMA

I. R. Smirnovskii

UDC 533.951

The shock-wave structure in an isotropic plasma is studied in the scale of the Debye shielding radius. The boundary condition at "infinity" is due to the dissipation mechanism of ions reflected from the ion wavefront. The case of the absence of dissipation corresponds to a collisionless shock wave. It is known that there is a critical Mach number above which the laminar shock wave is impossible. In addition, two types of boundary conditions which occur if collisions in an equilibrium high- and low-ionized plasma are taken into account are considered. The reflected ions are assumed to be scattered by electrons in the first case, and the momentum of the reflected ions is assumed to be transferred to uncharged particles in the second case. The critical Mach number of a shock wave does not exist under the conditions of collision dissipation of a flux of reflected ions.

Nonequilibrium plasma states, which are due to the evolution of a shock wave (SW), cause the phenomena of preionization and luminescence of the gas [1–11]. In some cases, this is explained by the appearance of epithermal electrons and subsequent excitation of the particles by the impact of electrons [12–15]. The substantial increase in the electron temperature T_e can be caused by the increasing instability and turbulence [12]. However, the increase in T_e also can occur owing to electron scattering of a laminar flux of reflected ions [16].

It is known that the asymptotic form (for $t \to \infty$) of the initial perturbation in a rarefied nonisothermal ($T_e \gg T_i$) plasma is a collisionless shock wave [17–21]. The front parameters of such a SW are determined by the "competition" of dispersion and nonlinearity effects. With absolutely cold ions $T_i = 0$ (no Landau damping), we have a solitary wave (soliton) before and after which the plasma state turns out to be the same [18, 20]. The soliton amplitude depends on its velocity, which is characterized by the Mach number $M_s = c/V_s$ (c is the wave velocity of a relatively undisturbed plasma, $V_s = \sqrt{T_e/m_i}$ is the velocity of the ion sound, and m is the mass). When the Mach number reaches the critical value M_s^0, the kinetic energy of bombarding particles is not sufficient to overcome the potential barrier. The ion density at the point with a maximum potential tends to infinity. A similar singularity is likely connected with the choice of the law of ion distribution in the form of a δ function.

It was shown in [21, 22] that M_{s*} for a SW is larger than the critical Mach number for a soliton ($M_{s*} \approx 1.82$). Bardakov et al. [22] showed the existence of a laminar SW for $M_s^0 < M_s < M_{s*}$. This is associated with the effect of the foot of a potential formed by a flux of ions reflected from the barrier. For $M_s = M_{s*}$, the ion density remains finite everywhere, and the number of reflected ions at the foot far from the front is equal to the number of incident ions.

The results of studies dealing with stationary perturbations in a nonisothermal plasma are of interest for the structure of shock waves in an equilibrium plasma. Ledenev [16] showed that the profile of a sufficiently strong SW in a high-ionized plasma includes the collisionless shock (CS) of the potential, and the necessary nonisothermal character of a plasma is ensured owing to energy dissipation of ions reflected by electrons.

Below, we shall consider the possibility of occurrence of a CS in a low-ionized equilibrium plasma. Perturbation of the fields of the charged component before a viscous shock of the neutral component, i.e., the

---

St. Petersburg State University, St. Petersburg 198904. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 39, No. 3, pp. 14–21, May–June, 1998. Original article submitted July 26, 1996.

0021-8944/98/3903-0335 $20.00

precursor (the leader), was investigated in [4-7, 23-27]. In the supersonic case ($c > V_s$), in particular, for an isothermal plasma, the solution is discontinuous in a long-wave approximation [26, 27]. The losses caused by ion–neutral particle collisions cannot prevent wave breaking. Therefore, in a plasma with an inviscid charged component the discontinuity structure can be determined by a CS whose spatial scale is much less than the length of the free path of ions.

We shall consider a nonequilibrium free plasma with the Maxwellian velocity distribution of electrons, in which the length of the free path of ions considerably exceeds the Debye shielding radius r_D. With allowance for the thermal motion of ions, the field structure on the spatial scale of the order of r_D is described using Moiseev and Sagdeev's stationary quasipotential method [17]. For the potential $\Phi = e\varphi/T_e$, the following equation [22] was derived:

$$\frac{d^2\Phi}{d\xi^2} = \nu_e(\Phi) - \nu_i(\Phi). \tag{1}$$

Here $\xi = r/r_D$ is the dimensionless coordinate in a reference system related to a maximum of the CS potential Φ_A ($\xi < 0$ in the domain before the CS front) and $r_D = \sqrt{T_e/4\pi n_0 e^2}$. The subscript A denotes the amplitude values of the fields in a CS for $\xi = 0$, ν_i is the ion concentration referred to the unperturbed value of n_0,

$$\nu_i(\Phi) = \nu_f \left(\int\limits_{\sqrt{2(\Phi-\Phi_f)}}^{\sqrt{2(\Phi_A-\Phi_f)}} F(W) \frac{W}{\sqrt{W^2 - 2(\Phi - \Phi_f)}} dW + \int\limits_{\sqrt{2(\Phi-\Phi_f)}}^{\infty} F(W) \frac{W}{\sqrt{W^2 - 2(\Phi - \Phi_f)}} dW \right), \quad \xi < 0,$$

$$\nu_i(\Phi) = \nu_f \int\limits_{\sqrt{2(\Phi_A-\Phi_f)}}^{\infty} F(W) \frac{W}{\sqrt{W^2 - 2(\Phi - \Phi_f)}} dW, \qquad \xi \geqslant 0,$$

$$F(W) = \exp\left(-(W - \mathrm{M}_f)^2/\beta\right)/\sqrt{\pi\beta}, \qquad W = \sqrt{v^2/V_s^2 + 2(\Phi - \Phi_f)}.$$

Here the subscript f refers to the values of the fields at the foot far from the front, $\mathrm{M}_f = v_f/V_s$ is the Mach number of the flux of ions incident on the barrier, ν_f is their concentration, v is the velocity; $\beta = 2T_i/T_e \ll 1$, ν_e is the dimensionless electron concentration, and $\nu_e = \exp(\Phi)$. Thus, there are two, incident and reflected, fluxes of ions at the foot for $\Phi = \Phi_f$. The distribution function of the ions in the incident flux is assumed to be Maxwellian with temperature T_i.

Equation (1) reduces to the form

$$\left(\frac{d\Phi}{d\xi}\right)^2 \Big/ 2 = -U(\Phi), \tag{2}$$

$$-U(\Phi) = \exp(\Phi) + \nu_f \left(\int\limits_{\sqrt{2(\Phi-\Phi_f)}}^{\sqrt{2(\Phi_A-\Phi_f)}} F(W) W \sqrt{W^2 - 2(\Phi - \Phi_f)}\, dW + \int\limits_{\sqrt{2(\Phi-\Phi_f)}}^{\infty} F(W) W \sqrt{W^2 - 2(\Phi - \Phi_f)}\, dW \right) + C, \quad \xi < 0,$$

$$-U(\Phi) = \exp(\Phi) + \nu_f \int\limits_{\sqrt{2(\Phi_A-\Phi_f)}}^{\infty} F(W) W \sqrt{W^2 - 2(\Phi - \Phi_f)}\, dW + C, \quad \xi \geqslant 0.$$

Here the integration constant

$$C = -\nu_f(M_f^2 + \beta/2) - \exp(\Phi_f) - \nu_f \int_0^{\sqrt{2(\Phi_A-\Phi_f)}} F(W)W^2\,dW$$

is found from the condition $U(\Phi_f) = 0$ and the requirement of $U(\Phi)$ continuity for $\xi = 0$. At the foot far from the CS front, owing to the quasineutrality we have the equation

$$\nu_r + \nu_f = \exp(\Phi_f), \tag{3}$$

where $\nu_r = \nu_f \int_0^{\sqrt{2(\Phi_A-\Phi_f)}} F(W)\,dW$ is the concentration of the reflected ions.

The solution of (2) depends on the plasma state at the foot, which, in turn, is determined by the mechanism of dissipation of the flux of reflected ions. We shall consider three idealized situations: (A) refers to the initially nonisothermal plasma, and (B) and (C) refer to the cases where the undisturbed plasma state is equilibrium and $T_e \gg T_i$ only in the heating zone before the shock wave owing to the electron thermal conductivity. The spatial scale of the heated zone is assumed to be much larger than the relaxation length of the ions in the reflected flux.

(A) Nonisothermal Collisionless Plasma. We shall ignore the energy dissipation of the particles in the reflected flux. We have a "running-away"-from-the CS isoimpulse foot, the velocity of the leading edge of which is equal to the velocity of the reflected ions. The profile of such a wave is not strictly stationary. However, the fields depend only on the spatial variable ξ in the neighborhood of a CS. Using a reference system related to the foot front, for the laws of conservation of the number of particles and energy, the following relation [22] was derived:

$$M_s + M_f = 2\nu_f M_f, \qquad (M_s^2 + M_f^2)/2 = \Phi_f + (2M_f)^2/2. \tag{4}$$

(B) Close-to-Equilibrium High-Ionized Plasma. We shall assume that the energy of the reflected ions completely converts to the thermal energy of electrons owing to electron–ion collisions and collective effects. The velocity of the foot's leading edge is equal to the SW velocity, and the states before and behind the CS are self-consistent. The problem of the SW structure in an equilibrium plasma without participation of the viscosity was formulated by Ledenev [16]. In such an SW, ion heating occurs owing to heat exchange between the electron and ion components. Hence, the electron temperature in the neighborhood of the CS density should be approximately twice the temperature obtained from the discontinuity relations. We have the laws of conservation of the number of particles, momentum, and energy in an undisturbed plasma-to-foot transition in the form [16]

$$M_s = M_f(\nu_f - \nu_r), \quad M_s^2 = \exp(\Phi_f) + M_f^2(\nu_f + \nu_r), \quad M_s^2/2 = M_f^2/2 + \Phi_f + 1. \tag{5}$$

(C) Low-Ionization Plasma. Energy scattering of the ions reflected by electrons in such a plasma may be ignored.

This is an essentially new situation, because, as far as we know, the problem of CS formation in a low-ionization plasma has not yet been formulated.

We assume that the ion flux decelerates mainly because of collisions with neutral particles, and ion heating occurs in a viscous shock wave of the neutral component.

We ignore the SW structure (the field distribution is specified as a "step") and omit the inverse action of the perturbation of the plasma component on the SW parameters. Then, using the known expression for the temperature T_2 behind the SW front and the equality $T_e = T_2$, we find

$$M_s^2 = \frac{(\gamma+1)^2}{2(\gamma-1)}\left[\left(1+\frac{2q}{\gamma-1}\right)\left(1-q\,\frac{\gamma-1}{2\gamma}\right)\right]^{-1}, \tag{6}$$

where $q = a^2/c^2$ is the SW intensity, a is the velocity of sound in an undisturbed gas, and γ is the adiabatic exponent.

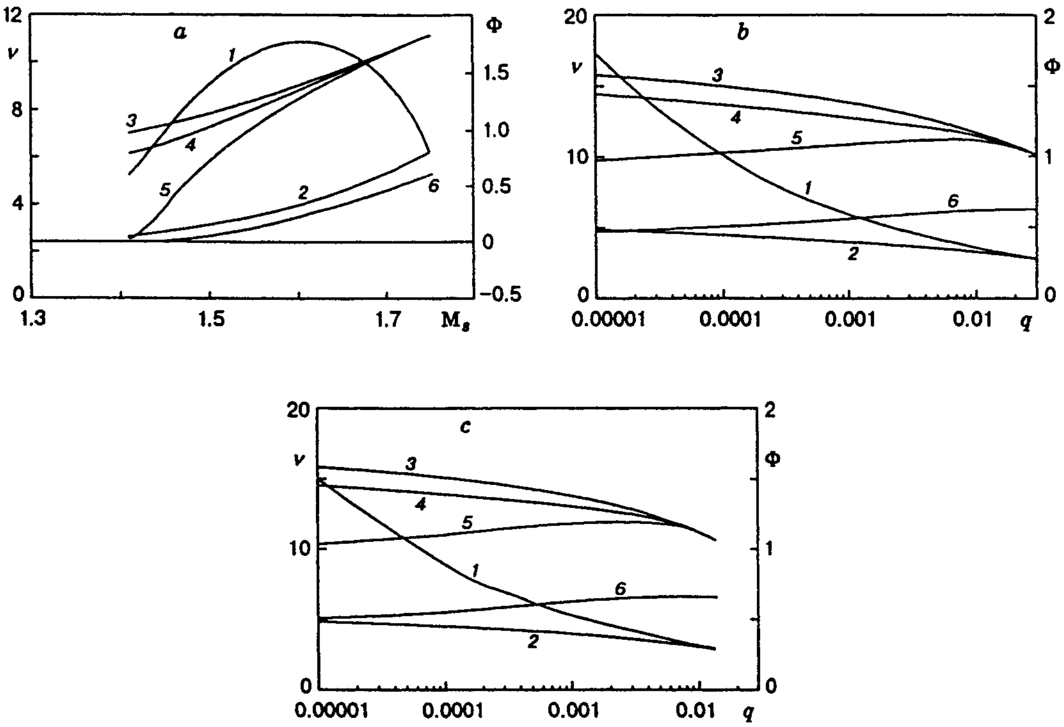

Fig. 1. Dependences of ν_{iA}, ν_{eA}, Φ_A, $\langle\Phi\rangle$, Φ_-, and Φ_f (curves 1–6) on the SW intensity for three cases of the dissipation of a flux of reflected ions: a) no dissipation [case (A)]; b) scattering by electrons [case (B)]; and c) scattering by neutral particles [case (C)]; to an increase of the intensity corresponds the growth of M_s (a) and a decrease in q (b) and (c).

We shall represent the laws of conservation of the number of particles, momentum, and energy at the foot as follows:

$$M_s = M_f(\nu_f - \nu_r), \quad M_s^2\left(1 + \frac{N_n \Delta v_n}{n_0 c}\right) = (M_f^2 + 1)(\nu_f + \nu_r),$$
$$\frac{M_s^2}{2}\left(1 + \frac{2N_n \Delta v_n}{n_0 c}\right) = \frac{M_f^2}{2} + \Phi_f + 1, \tag{7}$$

where Δv_n is the variation of the mean velocity of the neutral component, which results from the momentum transfer of the reflected ions, and N_n is the background concentration of the neutral component. In deriving (7), we have ignored the change in the energy of the neutral component compared with the change in the momentum, since $n_0/N_n \ll 1$ and $N_n \Delta v_n / n_0 c \sim 1$.

The roots of the equations

$$U(\Phi) = 0, \tag{8}$$

$$dU/d\Phi = 0 \tag{9}$$

describe the basic parameters of the CS. Equation (8) becomes an identity for $\Phi = \Phi_f$ and $\Phi = \Phi_A$ if $\xi < 0$ and for $\Phi = \Phi_-$ and $\Phi = \Phi_A$ (Φ_- is the minimum value of the oscillating potential behind the CS) if $\xi > 0$. We denote the solution of Eq. (9) in the domain $\xi > 0$ by $\langle\Phi\rangle$. It is noteworthy that $\langle\Phi\rangle$ is the value of the potential at the singular point of the differential equation (2), which corresponds to the "equilibrium" position when oscillations behind the CS are absent. Figures 1 and 2 show calculation data for the three cases of dissipation of the ion energy in the reflected flux [the boundary conditions (3)–(7)] and $\gamma = 5/3$.

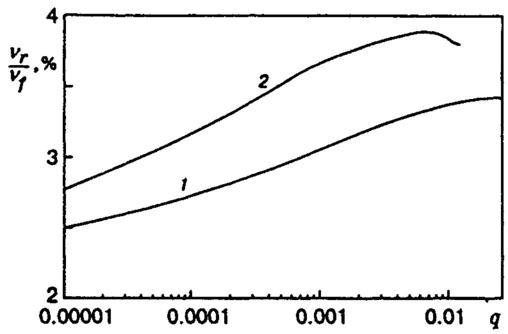

Fig. 2. Relative concentration of the reflected ions ν_r/ν_f at the foot versus the SW intensity q for two cases of dissipation of the flux of reflected ions: curves 1 and 2 refer to cases (B) and (C), respectively.

The results obtained by Bardakov et al. [22] for a collisionless SW in a free plasma are shown to be valid. In this case, the SW shape is determined by two independent parameters: the Mach number M_s and the degree of nonisothermality β. For fixed β, the SW intensity is characterized by M_s. The curves for the corresponding quantities at $\beta = 10^{-4}$ are shown in Fig. 1a.

There are regular oscillations of the fields behind the CS front on a spatial scale much smaller than the length of the free path of ions. The range of oscillations of the potential is characterized by the difference $\Phi_A - \Phi_-$ (Fig. 1). In case (A), with decrease in the shock-wave intensity M_s the depth of oscillations increases, and $\Phi_- \rightarrow 0$. The shock wave "degenerates" to a soliton. The collisions give rise to damping of oscillations and to reaching the "equilibrium" level of the fields, i.e., the potential tends to the value $\langle\Phi\rangle$ (Fig. 1).

The quantity $\langle\Phi\rangle$ determines the position of the discontinuity of a long-wave approximation, which describes plasma perturbation on spatial scales approximately equal to the length of the free path of ions. To calculate $\langle\Phi\rangle$, the self-consistency of the energy of regular plasma oscillations behind the CS and the plasma state at the foot makes it necessary to apply a procedure similar to that described in the present work.

In case (A) (Fig. 1a), the critical Mach number M_{s*} limits "from above" the intensity of a laminar collisionless SW for the formation of which the energy of the flux of reflected ions becomes insufficient. In a collisionless plasma (Fig. 1b and c), the situation is the opposite: a CS is formed no matter how large the intensity of the SW ($q \rightarrow 0$). The point is that, in case (A), the reflected ions interact with the wave once, carrying away the energy "to infinity." Localized at finite distances from the CS region of dissipation of the reflected flux, the ions can undergo multiple reflection from the wavefront, thus increasing the effectiveness of the damping mechanism based on the "capture" of a portion of particles by the wave.

For cases (B) and (C), there is a critical value of q_* above which a CS is not formed. For $q = q_*$, the potential profile in the CS becomes monotone, $\nu_{iA} = \nu_{eA}$, and $\Phi_A = \Phi_-$. In the scattering of the flux by electrons [case B], we obtain the value $q_* \approx 0.027$, which coincides approximately with the more approximate estimate from [16]. In this paper, without allowance for the viscosity, the CS parameters were apparently calculated in a broad range of q for the first time. In front of a viscous shock, the CS disappears in the neutral component [case (C)] at a smaller value of the critical parameter ($q_* \approx 0.0125$).

The appearance of q_* is caused by a decrease in the amount of dissipated energy transferred by the reflected flux, with a decrease in the SW intensity. The plasma states before and after the CS are related to the Hugoniot adiabat with allowance for the contribution of plasma oscillations behind the CS front. Here the latter is determined by the amount of energy carried away by the reflected ions. With decrease in the SW intensity, the laminar flux does not ensure the necessary level of dissipation, and the turbulence of the flux should be taken into account.

The critical intensity q_* is determined by the degree of plasma nonisothermality β, because, for $q = q_*$, β is approximately the same in cases (B) and (C). The small difference is, apparently, explained by a simplified idea of the inverse action of field perturbations before the SW front on the equilibrium values of the fields behind the SW front. In case (B), it is the electron temperature $T_e = 2T_2$ in the neighborhood of a CS, and, in case (C), it is the neglect of the inverse action of the charged component on the neutral one.

In practice, the ion flux decelerates owing to the interaction with both uncharged particles and electrons. Then, for an inviscid SW [case (B)], the quantity q_* will decrease because a fraction of the energy is spent for "friction" with neutral particles. In contrast to this, for a SW in a low-ionization plasma (case C), the quantity q_* will increase, because a portion of the energy of reflected ions converts to the heat energy of electrons. Thus, the parameters of real CS lie between the corresponding curves in Figs. 1b and c and 2.

For cases (B) and (C), the dependences of the concentration of the reflected ions on the intensity q are very different (curve 1 and 2 in Fig. 2). The growth of ν_r/ν_f in approaching q_* ensures the necessary level of dissipation owing to an increase in the number of reflected particles and, hence, momentum transfer. The subsequent drop of ν_r/ν_f is explained by the decrease in the potential barrier Φ_A. In case (B), the required magnitude of dissipation is ensured by a monotone growth of ν_r/ν_f and an increase in the internal energy of the electrons.

In concluding, it is necessary to emphasize that the notions of the quasiequilibrium function of electron distribution are not adequate to the real physical situation occurring at the foot in the dissipation zone of the flux of reflected ions. In such a description, the processes of relaxation of epithermal electrons, which are accompanied by ionization and excitation of the internal degrees of freedom of the particles, remain "beyond a frame." The mechanism of T_e increase, associated with these effects, differs substantially from the "action" of the electron thermal conductivity, which describes only small deviations of the electron distribution function from the locally equilibrium function.

Thus, the sufficiently strong initial perturbations in an equilibrium collision inviscid plasma evolve in the form of laminar SW. In a low-ionization gas, the formation of a plasma precursor before the SW front is possible. The characteristic feature of such perturbations is the appearance of epithermal electrons in a fairly extended region before the SW front. This is indirectly supported by luminescence and an increase in the degree of ionization of the ambient gas.

The author expresses his gratitude to V. A. Pavlov for his attention to the work.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 96-05-64723).

REFERENCES

1. G. I. Maikapar (ed.), *Nonequilibrium Physicochemical Processes in Aerodynamics* [in Russian], Mashinostroenie, Moscow (1972).
2. B. S. Punkevich and B. M. Stepanov, "Ionization of shock-unperturbed argon in an explosion of an HE charge," in: *Physical Processes in Combustion and Explosion* [in Russian], Atomizdat, Moscow (1980), pp. 151–155.
3. B. S. Punkevich and B. M. Stepanov, "Phenomenon of the initial flash of argon initiated by detonation of an HE charge," *ibid.*, p. 166–169.
4. I. V. Basargin and G. I. Mishin, "Propagation of a shock wave in a plasma of a transverse glow discharge in argon," *Pis'ma Zh. Tekh. Fiz.*, **11**, No. 4, 209–215 (1985).
5. V. A. Gorshkov, A. I. Klimov, G. I. Mishin, et al., "Specifics of the behavior of the electron density in a low-ionization nonequilibrium plasma during propagation of a shock wave in it," *Zh. Tekh. Fiz.*, **57**, No. 10, 1893–1898 (1987).
6. A. I. Klimov and G. I. Mishin, "Interferometric studies of shock waves in a gas-discharge plasma," *Pis'ma Zh. Tekh. Fiz.*, **16**, No. 24, 89–94 (1990).
7. G. I. Mishin, A. I. Klimov, and A. U. Gridin, "Shock-wave pressure and density measurements in a gas-discharge plasma," *Pis'ma Zh. Tekh. Fiz.*, **17**, No. 16, 84–89 (1991).

8. C. Covault, "Complex SDI Shuttle mission images aurora rocket plumes," *Aviation Week and Space Tech.*, **134**, No. 18, 18–21 (1991).
9. P. A. Bernhardt, W. E. Swartz, M. C. Kelly, et al., "Spacelab-2 upper atmospheric modification experiment over Arecibo, *Astrophys. Lett. Commun.*, **27**, No. 3, 183 (1988).
10. G. Holmagren, R. Bostrom, M. C. Kelley, et al., "Trigger, an active release experiment that stimulated auroral particle precipitation and wave emissions," *J. Geophys. Res.*, **85**, No. 10, 5043–5053 (1980).
11. N. F. Blagoveshchenskaya, V. N. Borodkin, Yu. V. Kolosov, et al., "Effects of chemical modification of the ionosphere based on data on short-wave Doppler measurements on sloping radiolines," *Geomagnetizm Aéronomiya*, **32**, No. 6, 122–127 (1992).
12. E. V. Mishin, Yu. Ya. Rushin, and V. A. Telegin, *Interaction of Electron Fluxes with the Earth's Ionosphere* [in Russian], Gidrometeoizdat, Leningrad (1989).
13. A. Yu. Ol'khovatov, "The role of epithermal electrons in the formation of luminous zones in the neighborhood of a space body," *Geomagnetizm Aéronomiya*, **30**, No. 1, 161–163 (1990).
14. A. Yu. Ol'khovatov, "Intensity of ionospheric plasma turbulence generated by a cosmic body," *Geomagnetizm Aéronomiya*, **32**, No. 4, 64–68 (1992).
15. A. Yu. Ol'khovatov, "Problem of a head echo of meteors," *Geomagnetizm Aéronomiya*, **31**, No. 4, 750–751 (1991).
16. V. G. Ledenev, "Shock-wave structure in a plasma," *Prikl. Mekh. Tekh. Fiz.*, No. 2, 17–20 (1990).
17. S. S. Moiseev and R. Z. Sagdeev, "Collisionless shock waves in a plasma in a weak magnetic field," *J. Nucl. Energy, Part C*, **5**, No. 1, 43–49 (1963).
18. R. Z. Sagdeev, "Collective processes and shock waves in a rarefied plasma" in: M. A. Leontovich (ed.), *Problems of the Theory of Plasma* [in Russian], Atomizdat, Moscow (1964).
19. D. W. Forslund and J. P. Freiberg, "Theory of laminar collisionless shocks," *Phys. Rev. Lett.*, **27**, No. 18, 1189–1192 (1971).
20. S. G. Alikhanov, V. G. Belan, G. N. Kichigin, and P. Z. Chebotaev, "A study of ion shock waves in a collisionless plasma," *Zh. Éksp. Teor. Fiz.*, **60**, No. 3, 982–992 (1971).
21. A. I. Akhiezer (ed.), *Electrodynamics of Plasma* [in Russian], Nauka, Moscow (1974).
22. V. M. Bardakov, A. G. Morozov, And I. G. Shukhman, "Structure of laminar shock waves in a collisionless plasma," *Fiz. Plazmy*, **1**, No. 6, 955–962 (1975).
23. R. F. Avramenko, A. A. Rukhadze, and S. F. Teselkin, "Shock-wave structure in a low-ionization nonisothermal plasma," *Pis'ma Zh. Éksp. Teor. Fiz.*, **34**, No. 9, 485–488 (1981).
24. V. A. Pavlov, A. N. Pinegin, and I. R. Smirnovskii, "Evolution of plasma perturbation in the *F* region and estimation of the ionosphere parameters on reciprocating probing data," *Izv. Vyssh. Uchebn. Zaved., Radiofiz.*, **36**, Nos. 3 and 4, 203–215 (1993).
25. V. A. Pavlov and I. R. Smirnovskii, "Structure of an ion-sonic wave in a point explosion," *Prikl. Mekh. Tekh. Fiz.*, **34**, No. 5, 3–9 (1993).
26. V. A. Pavlov and I. R. Smirnovski, "The ion sound precursor in low-ionized nonisothermal plasmas," *Abstracts XXIV Gener. Assemb. of URSI*, Kyoto, Japan (1993).
27. V. A. Pavlov, "Formation and structure of a ion-sonic shock wave in a low-ionization plasma," *Fiz. Plazmy*, **18**, No. 10, 1368–1371 (1992).